Name: Solutions

Teacher/Class: ______________________

SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT TASK 1

DECEMBER 2009

MATHEMATICS

Time Allowed: **70 minutes**

Instructions:

- Write your name and class at the top of each page
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start **each** question on a **new** page.
- Diagrams unless otherwise stated are **not** to scale

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	TOTAL
/8	8	/8	/8	/8	/8	/8	/56

Question 1 (8 marks)

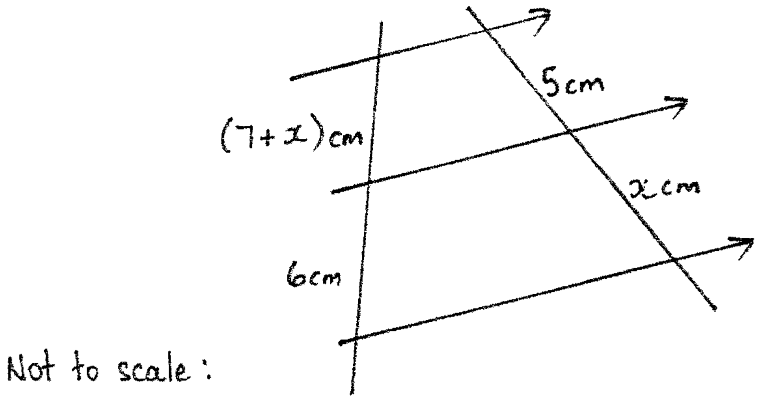
a) i) Solve $2x^2 - x = 1$ (3)

ii) Hence, solve $2x^2 - x - 1 > 0$

b) What is the vertex of $y = 3x^2 - 2x + 1$? (2)

c) What is the focal length of $x^2 = -12y$? (1)

d) Find the value of x in : (2)



Question 2 (8 marks) **Start a new page**

a) The roots of $5x^2 - 6x - 3 = 0$ are α and β (3)

i) Find the value of,

(I) $\alpha + \beta$

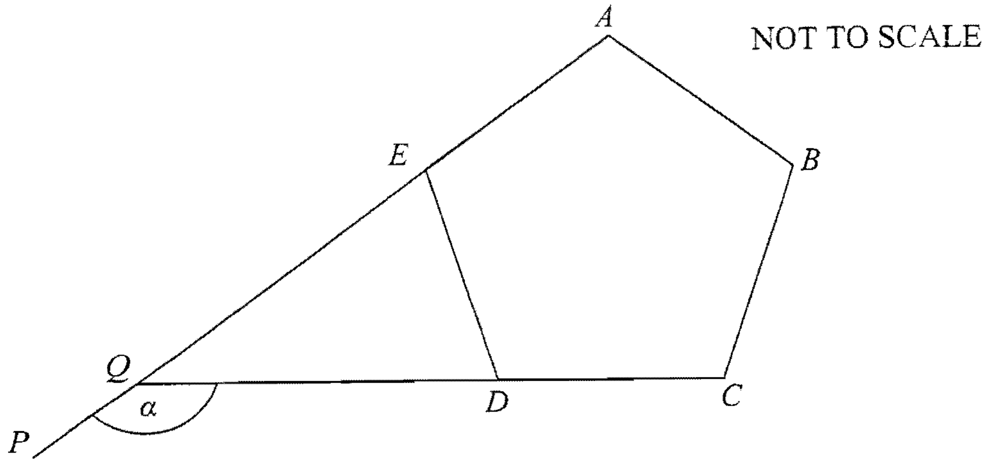
(II) $\alpha^2 \beta + \alpha \beta^2$

ii) Write down a quadratic equation, in the form $ax^2 + bx + c = 0$, where a, b and c are integers, whose roots are 2α and 2β. (3)

b) Find the locus of a point P which moves so that it is always 4 units from the point (-1,2) (2)

Question 3 (8 marks) **Start a new page**

a) ABCDE is a regular pentagon. The points P, Q, E and A are collinear.
CD is produced to meet PA at Q.



Find the size of angle α giving reasons. (2)

b) Consider the parabola $x^2 = 8y + 16$

i) Find the co-ordinates of the vertex (2)

ii) Find the co-ordinates of the focus (2)

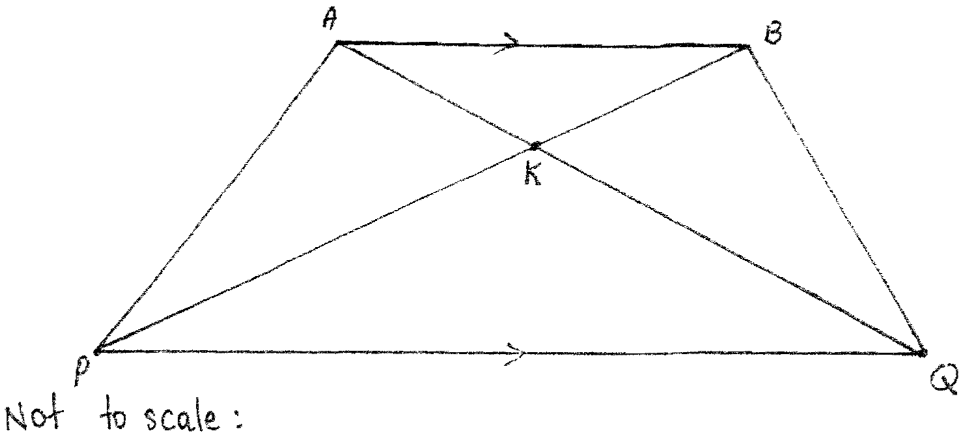
iii) Find the equation of the tangent to the parabola at the point $(2, -{}^{3}\!/_{2})$ (2)

Question 4 (8 marks) **Start a new page**

a) Find the radius and the centre of the circle whose equation is:

$$x^2 - 4x + y^2 + 6y - 12 = 0$$ (2)

b) In the diagram $AB \| PQ$,
AQ bisects PB at K and
PK = KQ



Copy the diagram onto your answer page, marking on it all relevant information.

i) Prove that Δ AKB is isosceles (3)

ii) Prove that $\Delta AKP \equiv \Delta BKQ$ (2)

iii) Hence, show that AP = BQ (1)

Question 5 (8 marks) **Start a new page**

a) Solve, $4^x - 9(2^x) + 8 = 0$ (3)

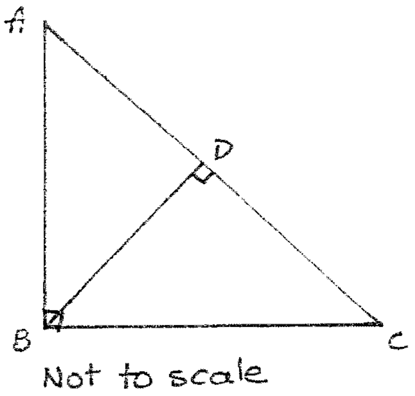
b) Find the value of A, B and C, given

$3x^2 + 4 \equiv A(x+2)^2 + B(x+2) + C$ (3)

c) Sketch the parabola $y^2 = 12x$ showing it's focus and directrix (2)

Question 6 (8 marks) **Start a new page**

a) ABC is a triangle in which $\angle ABC = 90° \text{ and } BD \perp AC$



i) Show that ΔABD is similar to ΔACB (2)

ii) If $AB = 3cm \text{ and } BC = 4cm$, find the length of BD, with reasons (2)

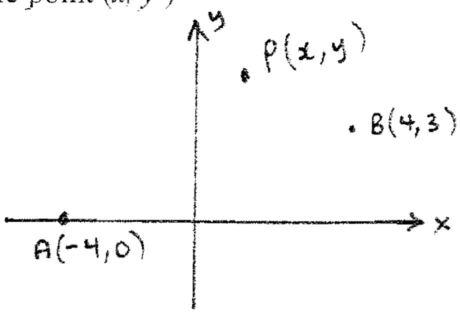
b) Consider the quadratic expression $3kx^2 - 5x + 3k$

i) Write down an expression for the discriminant of this quadratic. (1)

ii) Hence, find the value/s of k for which $3kx^2 - 5x + 3k$ is negative definite. (3)

Question 7 (8 marks) **Start a new page**

a) Let A and B be the fixed points (-4, 0) and (4, 3) respectively.
P is the variable point (x, y)



i) Write down expressions for PA^2 and PB^2 in terms of x and y (2)

ii) Hence, or otherwise, find the locus of the point P, such that, $PA = 2PB$ (3)

b) Find the value/s of k for which the quadratic equation,
$(k-3)x^2 - 3kx + 25 = 0$ has one root double the other root. (3)

End of paper

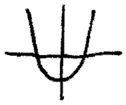
Answers

Question 1

a) $2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = -1/2, \; x = 1$

ii) 

$x < -\frac{1}{2}, \; x > 1$

b) $x = \frac{2}{6}$ Vertex $\left(\frac{1}{3}, \frac{2}{3}\right)$

c) $x^2 = -12y$

focal length 3 units

d) $\frac{x+7}{6} = \frac{5}{x}$

$x^2 + 7x = 30 \quad x > 0$

$(x+10)(x-3) = 0$

$\therefore x = 3$

Question 2.

a) $\alpha + \beta = -\frac{b}{a}$

$= \frac{6}{5}$

ii) $\alpha\beta(\alpha+\beta) = \frac{c}{a} \times \frac{6}{5}$

$= \frac{-3}{5} \times \frac{6}{5}$

$= \frac{-18}{25}$

ii. $x^2 - (2\alpha + 2\beta)x + 2\alpha . 2\beta = 0$

$x^2 - 2(\alpha+\beta)x + 4\alpha\beta = 0$

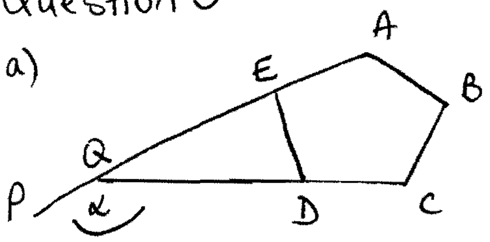
$x^2 - 2(6/5)x + 4(-3/5) = 0$

$5x^2 - 12x - 12 = 0$

b) $\sqrt{(x+1)^2 + (y-2)^2} = 4$

$\therefore (x+1)^2 + (y-2)^2 = 16.$

Question 3

a) 

$\angle AED = \angle CDE = 108°$

(angles of a regular pentagon)

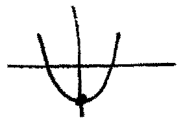
$\angle QED = \angle EDQ = 72°$ (2)

(straight line equals 180°)

$\alpha = \angle QED + \angle EDQ$ (exterior $\angle$ of ΔQED).

$\alpha = 144°$

b) $x^2 = 8(y+2)$ (2) $a = 2$



i. $V(0,-2)$ (2)

ii. $S(0,0)$ (2)

iii. $8y = x^2 - 16$

$y = \frac{x^2}{8} - 2$

$dy/dx = 2x/8$ at $x = 2$

$M_T = 1/2$

$\therefore y + 3/2 = 1/2(x-2)$

$2y + 3 = x - 2$

$2y = x - 5.$ (2)

Question 4

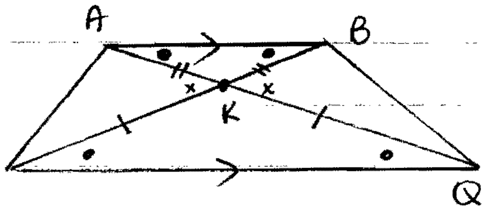
a) $x^2 - 4x + y^2 + 6y = 12$

$\quad +4 \qquad +9 \qquad +13$

$(x-2)^2 + (y+3)^2 = 25$

$C = (2, -3) \qquad r = 5$

b)



i. $\angle KPQ = \angle KQP$ (equal angles opposite equal sides, $PK = KQ$)

$\angle KBA = \angle KPQ$ (alt $\angle$'s $AB \parallel PQ$)

$\angle BAK = \angle KQP$ (alt $\angle$'s $AB \parallel PQ$)

$\therefore \triangle AKB$ is isosceles (2 equal $\angle$'s)

ii. In $\triangle AKP$ and $\triangle BKQ$

$PK = QK$ (given)

$AK = BK$ (equal sides opposite equal $\angle$'s, $\triangle AKB$ isosceles.)

$\angle AKP = \angle BKQ$ (vertically opposite)

$\therefore \triangle AKP \equiv \triangle BKQ$ (SAS)

iii. $AP = BQ$, corresponding sides in congruent triangles

Question 5

a) let $m = 2^x \therefore m^2 - 9m + 8 = 0$

$(m-8)(m-1) = 0$

$m = 8$ or $m = 1$

ie $2^x = 8$ or $2^x = 1$

and $x = 3$ or $x = 0$

b) $3x^2 + 4 = Ax^2 + 4Ax + 4A + Bx + 2B + C$

Equating coeff

$[x^2]$ $\quad 3 = A \qquad \underline{A = 3}$

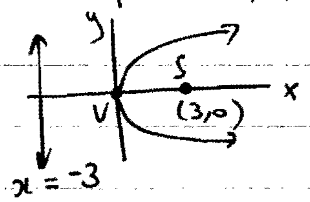
$[x]$ $\quad 0 = 4A + B$

$B = -12 \qquad \underline{B = -12}$

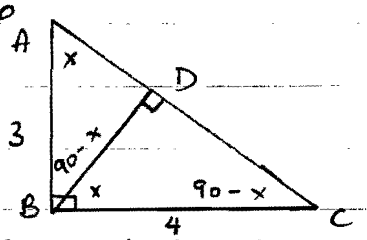
$[x^0]$ $\quad 4 = 4A + 2B + C$

$4 = 12 + -24 + C \qquad \underline{C = 16}$

c) Sideways $V = (0,0)$ $\quad a = 3$



Question 6



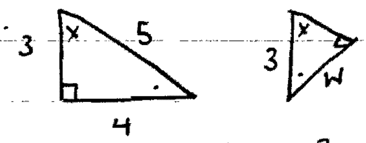
i. In $\triangle ABD$ and $\triangle ACB$

$\angle ADB = \angle ABC$ (given 90°)

$\angle A$ is common

$\therefore \triangle ABD \parallel\!| \triangle ACB$ (equiangular)

ii.



$\frac{w}{4} = \frac{3}{5}$ (ratio of corresp sides in $|||$ $\triangle$'s)

$\therefore 5w = 12$

$w = 2.4 \quad \therefore \underline{BD = 2.4\text{cm}}$

b) $3kx^2 - 5x + 3k = 0$

i. $\Delta = b^2 - 4ac$

$= 25 - 4(3k)(3k)$

$\Delta = 25 - 36k^2$

ii. $3k < 0$ and $\Delta < 0$

$k < 0 \qquad (5-6k)(5+6k) < 0$

$\therefore$ Sol^n: $k < -5/6$ only.

Question 7

a) i. $PA = \sqrt{(x+4)^2 + (y-0)^2}$

$PA^2 = (x+4)^2 + y^2$

$PB^2 = (x-4)^2 + (y-3)^2$

ii. $PA = 2PB$

$PA^2 = 4PB^2$

$x^2+8x+16+y^2 = 4[x^2-8x+16+y^2-6y+9]$

$x^2+8x+16+y^2 = 4x^2-32x+64 + 4y^2-24y+36$

$3x^2-40x+48+3y^2-24y+36=0$

$3x^2-40x+3y^2-24y+84=0$

b) let the roots be α and 2α

$\alpha+2\alpha = -b/a$ $\qquad$ $\alpha \times 2\alpha = c/a$

$3\alpha = \frac{3K}{K-3}$ $\qquad$ $2\alpha^2 = \frac{25}{K-3}$ — ②

$\alpha = \frac{K}{K-3}$ — ①

Sub ① into ②

$2\left[\frac{K}{K-3}\right]^2 = \frac{25}{K-3}$ $\qquad (K \neq 3)$

$2 \cdot \frac{K^2}{(K-3)^2} = \frac{25}{K-3}$ $\qquad \times (K-3)$

$\frac{2K^2}{K-3} = 25$

$2K^2 = 25(K-3)$

$2K^2 - 25K + 75 = 0$

$(2K-15)(K-5) = 0$

$\therefore K = 15/2$ and $K = 5$.